VIBRATION-EXTRUSION DISCHARGE OF DISPERSE SYSTEMS*

Z. E. Filer, B. A. Lishanskii,
Yu. L. Vorob'ev, and V. K. Presnyakov

UDC 665.015:691.15

The physical principles governing the vibration-extrusion discharge of disperse systems are discussed. The criterion for optimizing this process is obtained and methods for achieving it are indicated.

It is shown in [1-4] that for certain vibrational modes of a working device (container, trough, etc.) a free-flowing viscous material is separated from the bottom of the vibrating device with subsequent separate motions of the material and container.

In the present paper we examine the vibration-extrusion discharge from a container of disperse systems with viscous friction for the possibility of an effective technological use of this process.

It is known that when a vibrating container and a viscous material move separately there is a viscous interaction between them and a dissipation of energy of the material during its separation and interaction with the surrounding medium. Separate motion ends with an impact which in the first approximation can be considered as instantaneous and completely inelastic.

As a consequence of the large dissipation of energy in the material, wave processes initiated in it after impact are rapidly damped out.

It should also be noted that a shock wave is propagated rather rapidly in the vibrating viscous material, since the effect is transmitted at the speed of sound and the thickness of the layer of material is relatively small, and therefore we can assume that the total mass of material simultaneously makes an inelastic collision with the bottom of the container (Fig. 1).

After separation of the material an air cushion is formed between it and the bottom of the container. Because of the hole in the bottom of the container this cushion does not exert an appreciable elastic resistance during the time preceding the impact.

Material is discharged through the hole in the vibrating container.

If the acceleration of the vibrating system is greater than the acceleration due to gravity, there is an instantaneous decrease in the vibrating mass which is equal to the mass m_0 of the material which has "taken off" and an instantaneous increase in the mass of the "pulled down" material by the same amount when they collide.

The equation describing the vibrations of the system can be written in the form

$$m_1\ddot{x} + \beta\dot{x} + Cx = m_1 g + F(t), \tag{1}$$

where $m_1 = m + \sigma m_0$ becomes a step function of the law of motion sought, g is the acceleration due to gravity, and σ equals 1 for common motion and 0 for the separate motions of the container and material.

Equation (1) is nonlinear even without taking account of the effect of the impact, and m_1 becomes discontinuous during acceleration, having a first-order discontinuity when the separation and recombination of

*Paper presented at the Seventh Symposium on the Rheology of Polymers, Moscow, April 10-14 (1972).

Khar'kov House-Building Combine No. 1. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 30, No. 2, pp. 221-227, February, 1976. Original article submitted May 8, 1975.

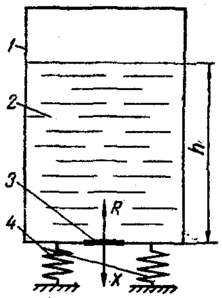

Fig. 1. Schematic diagram of vibration-extrusion discharge of disperse systems. 1) Container; 2) free-flowing or viscous material; 3) hole; 4) elastic couplings (springs or rubber bumpers).

mass of the vibrating material are idealized as instantaneous processes, since σ is a function of the required solution and its second derivative.

The instantaneous impact idealization also leads to a nonlinear effect of the delta function type with the strength and instant of the action determined by the vibration regime sought.

Assuming that the interaction is weak, the motion of the container can be considered sinusoidal in the first approximation, and the asymptotic method [5] can be used.

To obtain the equations of motion we use expressions for the kinetic energy $\bar{T}$, the potential energy V, and the dissipative function Φ which take account of the possibility of the separation of material:

$$\left.\begin{aligned} \bar{T} &= \frac{1}{2}\left[m\dot{x}^2 + \sigma m_0 \dot{x}^2 + m_0 \dot{x}_0^2 (1-\sigma)\right], \\ V &= \frac{1}{2} Cx^2 - \left[mq(x) + \sigma m_0 q(x) + m_0 q(x_0)(1-\sigma)\right], \end{aligned}\right\} \quad (2)$$

$$\Phi = \frac{1}{2}\left[\beta\dot{x}^2 + \beta_0(\dot{x} - \dot{x}_0)^2(1-\sigma) + \beta_1 \dot{x}_0^2(1-\sigma)\right]. \quad (3)$$

The instant of separation t_0 of material from the container is determined from the condition

$$\ddot{x}(t_0) = \frac{dq[x(t_0)]}{dx}, \quad (4)$$

after which the masses m and m_0 move separately until they collide, starting at the time t_1 when

$$x(t_1) = x_0(t_1). \quad (5)$$

The position $x(t_0)$ and velocity $\dot{x}(t_0)$ of the total mass at the instant of separation are the initial conditions for the motion of the mass m_0.

At the instant of impact the velocity of the container changes abruptly, as given by

$$\dot{x}(t_1 + 0) = \frac{m\dot{x}(t_1 - 0) + m_0 \dot{x}_0(t_1 - 0)}{m_1 + m_0}. \quad (6)$$

The initial conditions for the common motion stage are $x(t_1)$ and $\dot{x}(t_1 + 0)$, where $\dot{x}(t_1 + 0)$ is the velocity of the material after impact.

After impact and until the common motion ceases there will be a vibration-extrusion discharge of free-flowing or viscous material from a constant height h through the hole in the bottom of the container with the velocity

$$v_{\text{rel}} = \sqrt{2h\left[\frac{dq(x)}{dx} - \ddot{x}\right]}, \quad (7)$$

if the flow is laminar, i.e., if $\text{Re} < \text{Re}_{\text{cr}}$.

For turbulent flow when $\text{Re} > \text{Re}_{\text{cr}}$ the vibration-extrusion discharge velocity of the material will be

$$v_{\text{rel}} = \frac{1}{\varphi(\text{Re})}\sqrt{2h\left[\frac{dq(x)}{dx} - \ddot{x}\right]}, \quad (8)$$

where the function $\varphi(\text{Re})$ is found experimentally.

Optimization seeks to increase the absolute velocity of the vibration-extrusion discharge of material which, taking account of the technological features of the process, can enter a stationary or moving volume such as a form for the vibrational compaction of a concrete mixture.

In the case under consideration, achieving the required degree of compaction, i.e., the necessary density of the material, depends on the absolute velocity of discharge of the concrete mixture.

The absolute discharge velocity of the material is given by

$$v = \dot{x} + v_{\text{rel}} = \dot{x} + \sqrt{2h\left[\frac{dq(x)}{dx} - \ddot{x}\right]}, \tag{9}$$

where $\dot{x}$ is the velocity of the vibrations of the container.

In addition to the exciting force F(t) of the vibrator, the container is also subjected to the reaction R of the discharging stream of material given by the expression

$$Rdt = v_{\text{rel}}\,dm,$$

where dm is an element of mass. Hence,

$$R = 2S\rho h\left[\frac{dq(x)}{dx} - \ddot{x}\right]k\sigma, \tag{10}$$

where k is the contraction coefficient of the effluent stream.

Introducing the mass $\overline{m}$, which takes account of the reaction of the stream, in the form

$$\overline{m} = 2Sk\rho h, \tag{11}$$

we obtain the equations of motion of the container and material:

$$[m + (m_0 - \overline{m})\sigma]\,\ddot{x} + \beta\dot{x} + \beta_0(1-\sigma)(\dot{x} - \dot{x}_0) + Cx = F(t) + [m + (m_0 - \overline{m})\sigma]\,g_1; \tag{12}$$

$$[m_0\ddot{x}_0 + \beta_1\dot{x}_0 - \beta_0(\dot{x} - \dot{x}_0) - m_0 g_1](1-\sigma) = 0, \tag{13}$$

where

$$g_1 = \frac{dq(x)}{dx} \quad \text{(in the gravitational field } g_1 = g\text{)}.$$

For common motion $\sigma = 1$ and Eq. (13) becomes an identity,

$$x(t) \equiv x_0(t).$$

We choose the period T of the exciting force F(t) as the time standard and take g_1 as the standard of acceleration. Then the velocity standard will be g_1T and the length standard, g_1T^2.

Transformation to dimensionless variables gives

$$\tau = \frac{t}{T}, \quad y = \frac{x}{g_1T}, \quad y_0 = \frac{x_0}{g_1T^2}. \tag{14}$$

In terms of the new variables, Eqs. (12) and (13) become

$$[1 + (\xi - \xi_n)\sigma]\,y'' + \alpha y' + \alpha_0(y' - y_0')(1-\sigma) + \omega^2 y = f(\tau) + 1 + (\xi - \xi_n)\sigma; \tag{15}$$

$$(1-\sigma)[\xi y_0'' - \alpha_0(y' - y_0') + \alpha_1 y_0' - \xi] = 0, \tag{16}$$

where

$$\left.\begin{aligned} \xi &= \frac{m_0}{m}, \quad \xi_n = \frac{\overline{m}}{m}, \quad \alpha = \frac{\beta T}{m}, \\ \alpha_0 &= \frac{\beta_0 T}{m}, \quad \alpha_1 = \frac{\beta_1 T}{m}, \quad \omega^2 = \frac{CT^2}{m} \end{aligned}\right\}. \tag{17}$$

We determine the instant of separation τ_0 from the expression

$$y''(\tau_0) = 1 \tag{18}$$

in the same way as the instant of impact, τ_1 is determined from the equation

$$y(\tau_1) = y_0(\tau_1), \tag{19}$$

as its nearest root to the right of τ_0.

The initial velocity of the common motion of the container and material is

$$y'(\tau_1 + 0) = \frac{y'(\tau_1 - 0) + \xi y_0'(\tau_1 - 0)}{1 + \xi}. \tag{20}$$

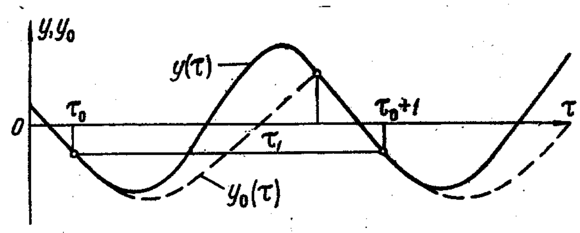

Fig. 2. Graph of the periodic solution of Eqs. (15) and (16) obtained by computer.

If the vibrations are excited by a harmonic force

$$F(t) = F_0 \sin \nu t \quad \text{and} \quad T \frac{2\pi}{\nu}, \tag{21}$$

then

$$f(\tau) = f_0 \sin 2\pi\tau, \tag{22}$$

where

$$f_0 = \frac{F_0}{mg_1}. \tag{23}$$

In this formulation of the problem the goal of the optimization is to increase the maximum discharge velocity for the function

$$J(\tau) = y'(\tau) + u_0 \sqrt{1 - y''(\tau)} \tag{24}$$

on the interval $\tau_1 < \tau < \tau_0 + 1$, where

$$u_0 = \sqrt{\frac{2h}{g_1 T^2}}. \tag{25}$$

If restrictions are imposed on the magnitude of the impact impulse, the inequality

$$\frac{\xi}{1+\xi} |y'(\tau_1 - 0) - y_0'(\tau_1 - 0)| < J_0. \tag{26}$$

must be satisfied, where the dimensionless quantity J_0 is related to the permissible value of the impact impulse J by the relation

$$J_0 = \frac{\dot{J}}{mg_1 T}. \tag{27}$$

Another restriction may be the guarantee of a given output

$$\int_{\tau_1}^{\tau_{01}} \sqrt{1 - y''(\tau)}\, d\tau > \dot{J}_1,$$

where $\dot{J}_1$ characterizes the amount of material discharged during a period of steady vibrations.

It is clear that the calculation of

$$V_{\max} = \max, \quad \max \dot{J}(t),$$
$$\xi,\ \xi_n,\ f_0,\ u_0, \quad \tau \in (\tau_1, \tau_0 + 1) \tag{28}$$

when (26) is satisfied is performed for steady vibrational regimes, i.e., when

$$y(\tau_0 + 1) = y(\tau_0),\ y_0(\tau_1 + 1) = y_0(\tau_1),$$
$$y'(\tau_0 + 1) = y'(\tau_0),\ y_0'(\tau_1 + 1 - 0) = y_0'(\tau_1 - 0). \tag{29}$$

Figure 2 shows a graph of the periodic solution of Eqs. (15) and (16) found numerically by computer.

Figure 2 shows that the impact impulse decreases with decreasing relative velocity of the container and material at collision; i.e., if the collision occurs before the maximum of $y(\tau)$, the velocities $y'(\tau)$ and $y_0'(\tau)$ are nearly the same.

The periodic solution of Eqs. (15) and (16) can be obtained by the method of matching, but the analytic solution is complicated, involving a series of transcendental equations, and requires finding the roots of a cubic equation; therefore, it is expedient to use an analog–digital complex to investigate the separate motions. This enables one to study different variants, to separate out the admissible ones, and to select the one closest to optimum. The initial conditions for the first stage are best chosen by finding an approximate solution by the asymptotic method [5], varying the parameters of the vibrating system (the stiffness of the elastic couplings and the amplitude of the exciting force) and the area of the hole, the depth of the layer of material, and the ratio of the masses of the material and the container.

It is convenient to use the method of gradient search to choose the initial parameters of the experimental–commercial sample of the vibration extruder to obtain the optimum variant.

It is also possible to choose the optimum exciting function $F(t)$ without presupposing that it is sinusoidal, but only periodic.

Thus, the problem can be formulated as a problem of parametric optimization, and more generally as a problem of optimum control. The optimization criterion chosen in (24) corresponds to the use of a vibration extruder for disperse systems such as concrete mixtures, where the useful effect is the pressure head of the emerging portion of the concrete mixture. This is helpful in filling a form for a concrete product [6].

A sample of a vibration extruder was tested on the Khar'kov DSK-1.

The experiments showed that the vibration-extrusion discharge of low-slump concrete mixtures can be used to increase the assembly-line production of reinforced-concrete products.

NOTATION

m_0, mass of material; m, mass of container; C, stiffness of elastic couplings; $F(t)$, exciting force; $\overline{T}$, kinetic energy; V, potential energy; Φ, dissipative function; $q(x)$ potential of field; β, β_0, B_1, equivalent coefficients of viscous resistance to the motion of container, material relative to container, and material relative to medium, respectively; x, x_0, absolute displacements of container and material; t_0, instant of separation of material from container; t_1, instant of collision of material with container; h, depth of material in container; Re, Reynolds number; v, absolute velocity of discharge of material; v_{rel}, relative velocity of discharge of material; $\dot{x}$, velocity of vibrating container; R, reaction to effluent stream of material; ρ, density of material; S, area of hole; $\overline{m}$, mass of material; k, contraction coefficient of stream; τ, y', y_0, dimensionless time, velocity, and displacement; T, period of vibrations; ν, circular frequency; dot over a quantity denotes its time derivative.

LITERATURE CITED

1. I. I. Blekhman and G. Yu. Dzhanelidze, Vibrational Transfer [in Russian], Nauka, Moscow (1964).
2. B. A. Lishanskii, N. V. Mikhailov, and P. A. Rebinder, Dokl. Akad. Nauk SSSR, 181, No. 6 (1968).
3. B. A. Lishanskii and B. P. Osmachkin, in: Rules for the Formation and Breakdown of Dispersed Materials [in Russian], Vol. 4 (edited by P. A. Rebinder and A. V. Lykov), Minsk, (1972), p. 207.
4. A. F. Ryzhkov and A. P. Baskakov, Inzh.-Fiz. Zh., 27, No. 1 (1974).
5. V. K. Presnyakov and Z. E. Filer, in: Development of Mineral Deposits [in Russian], Vol. 6, Tekhnika, Kiev (1966), p. 46.
6. Yu. L. Vorob'ev, N. V. Mikhailov, B. A. Lishanskii, and É. F. Lykashenko, Author's Certificate No. 293692; Byul. Izobret., No. 6 (1971).